Chapter 22: The Electric Field

Read Chapter 22

- **Do Ch. 22 Questions 3, 5, 7, 9**
- **Do Ch. 22 Problems 5, 19, 24**

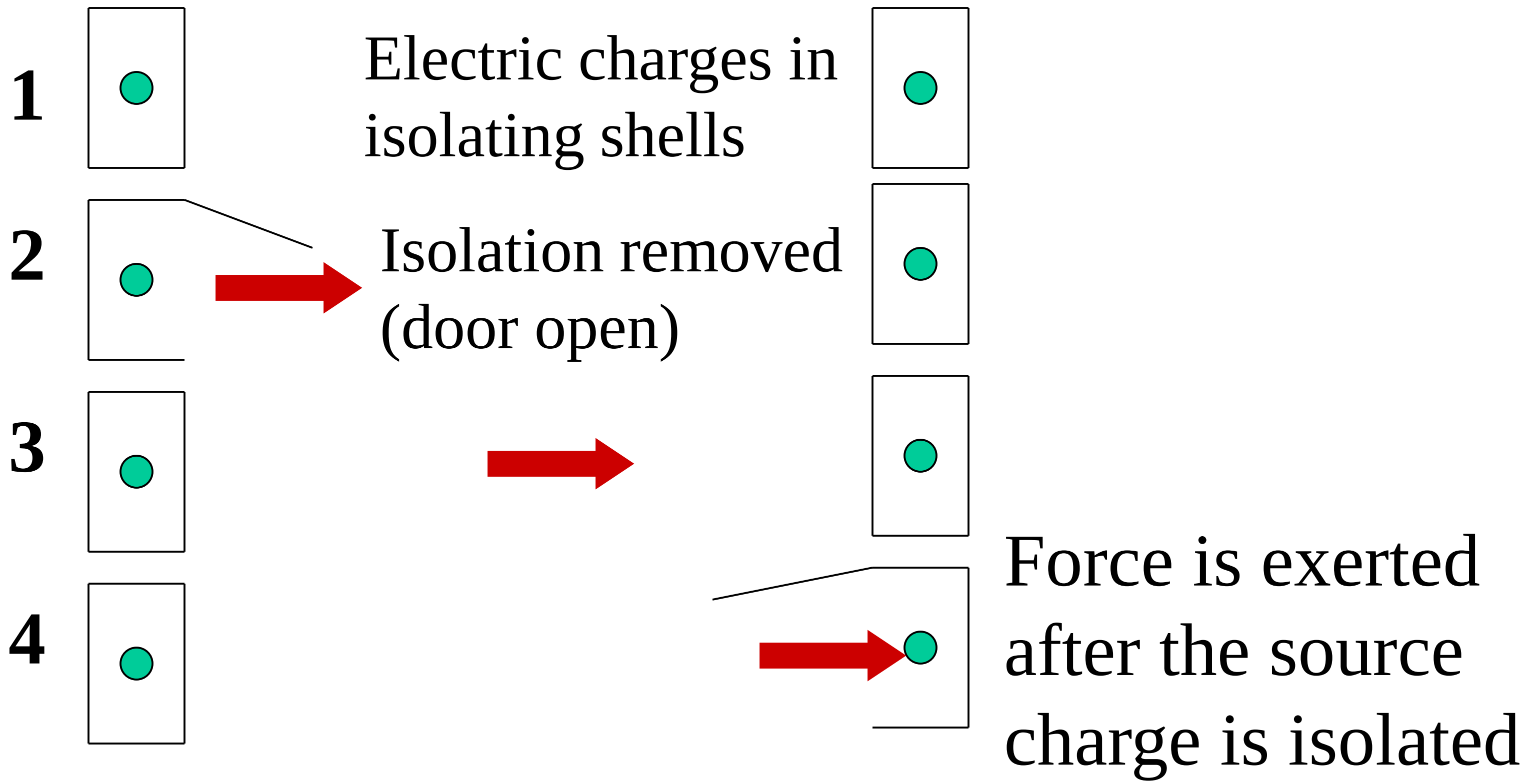
The Electric Field

- Replaces action-at-a-distance
- Instead of Q_1 exerting a force directly on Q_2 at a distance, we say:
- Q_1 creates a ***field*** $\vec{E}$ and then the ***field*** exerts a force on Q_2.
- NOTE: Since force is a vector then the electric field must be a ***vector field!***

$$\vec{F} = q\vec{E}$$

Does the field really exist?

It exists due to the finite speed of light (maximum speed of signal propagation)

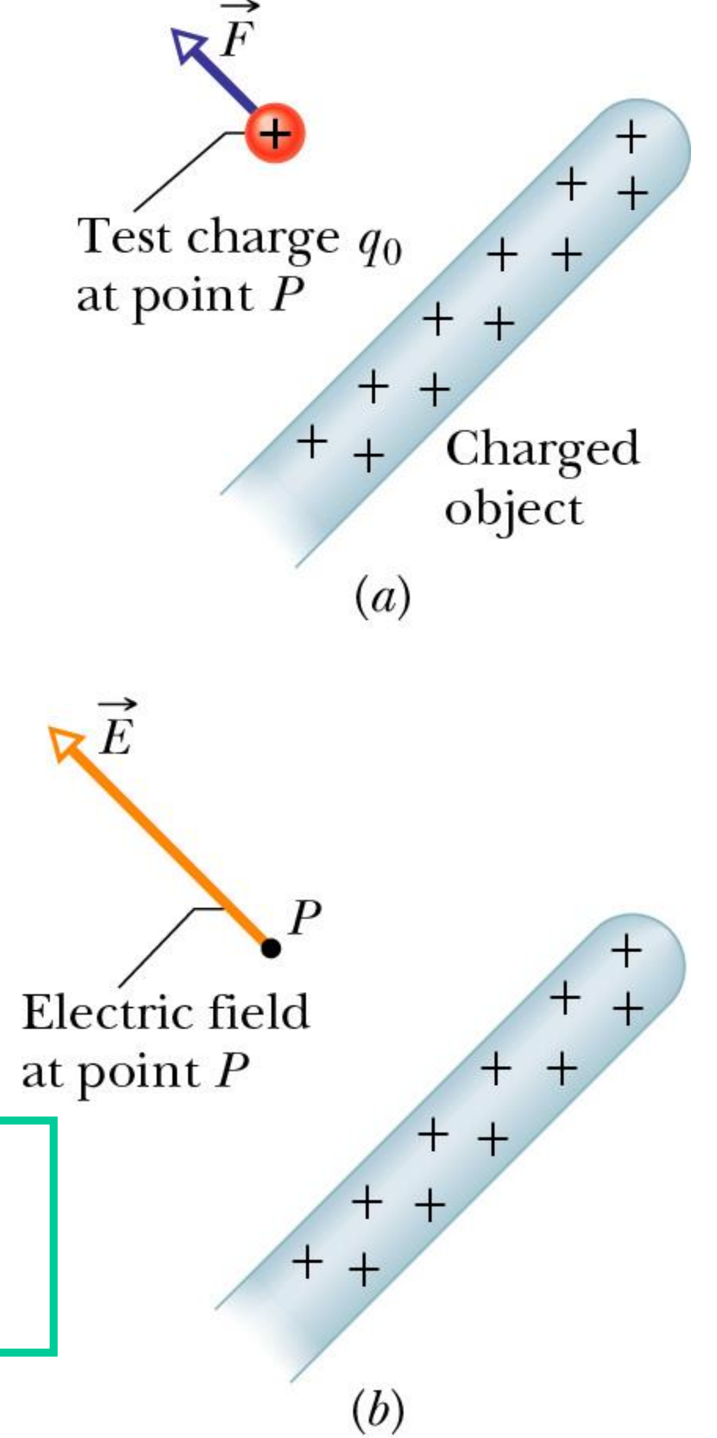


Interaction by the field rather than by charge

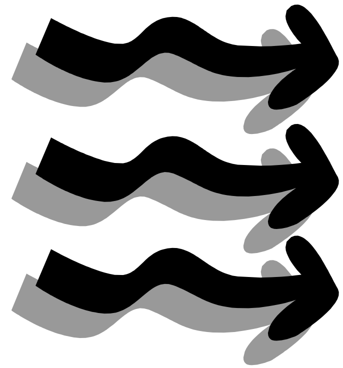
Field E is defined as the force that would be felt by a unit positive test charge

$$\vec{E} = \vec{F} / q_0$$

SI units for the electric field: newtons per coulomb.



Electric Field Lines

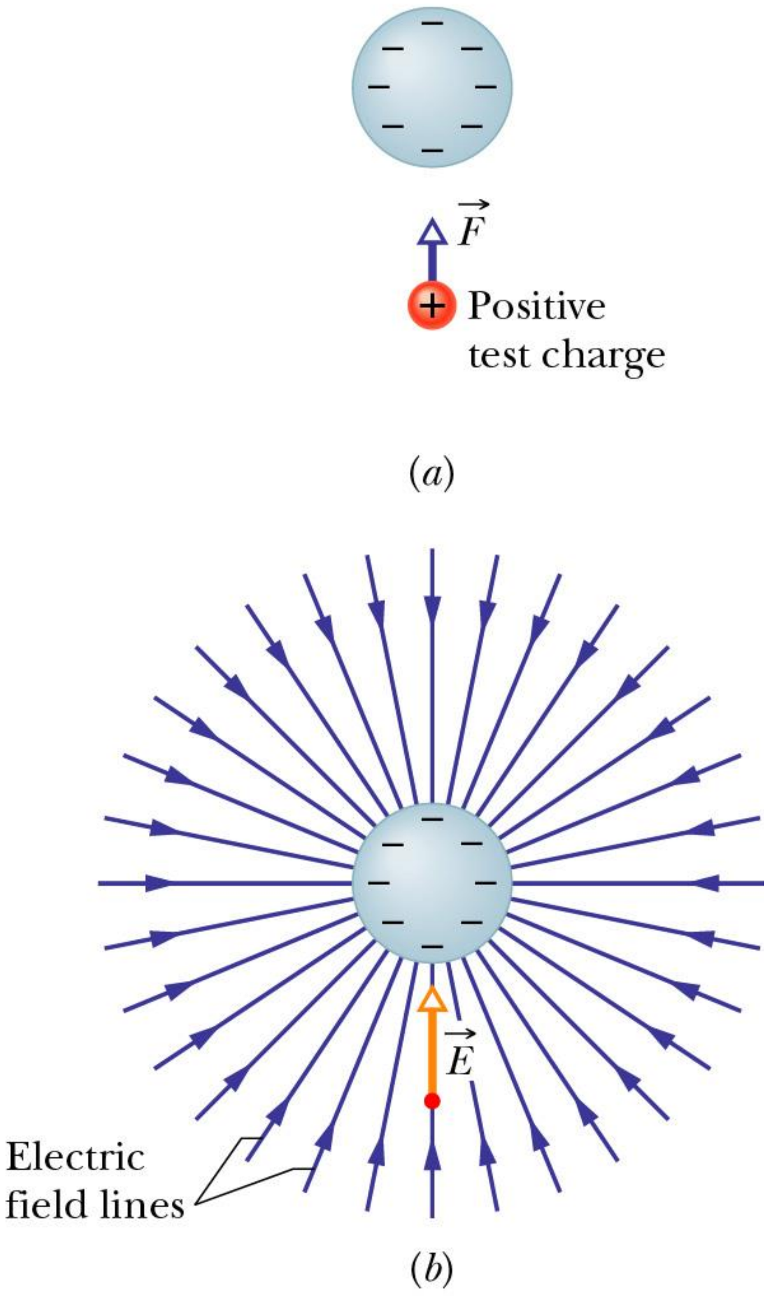


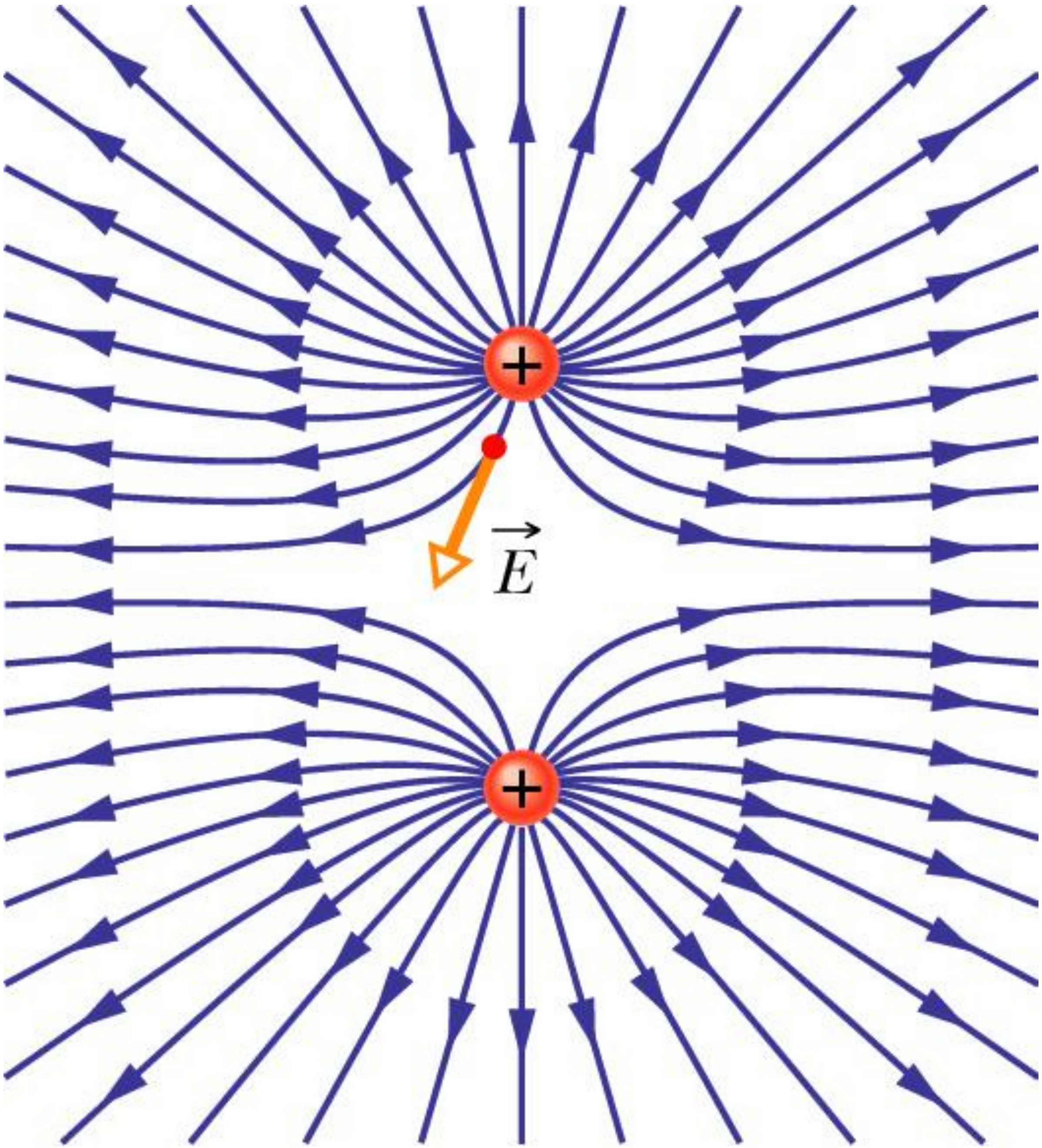
We **visualize** the field by drawing **field lines.**

These are defined by **three properties:**

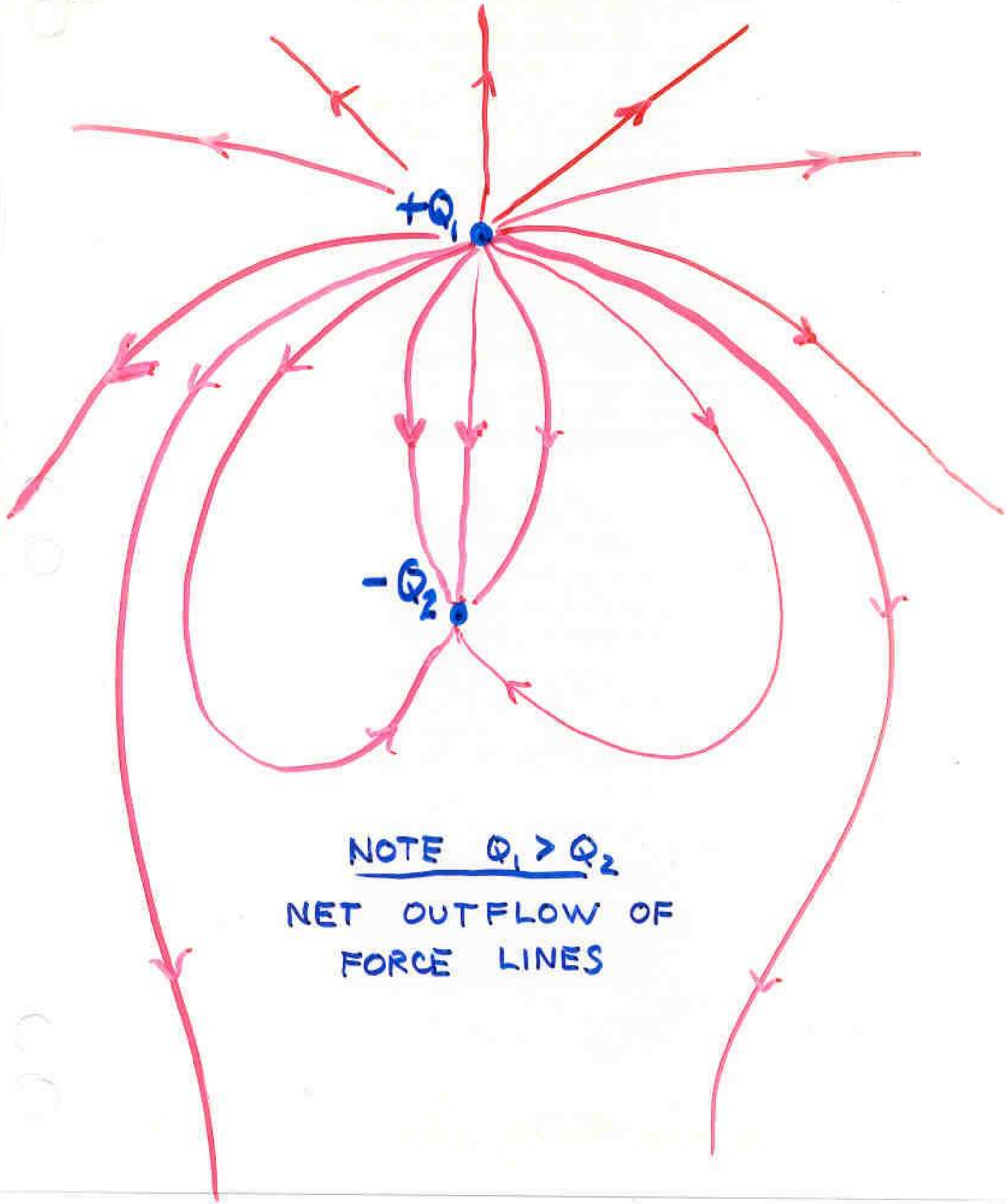
- Lines point in the **same direction** as the field.
- Density of lines gives the **magnitude** of the field.
- Lines begin on + charges; end on – charges.

Electric field created by a negatively charged metal sphere




$\vec{E}$

$\vec{E}$ FIELD DUE TO TWO POINT CHARGES



Coulomb's Law for the Field

Coulomb's law for the force on *q* due to *Q*:

$$\vec{F} = k\frac{qQ}{r^2}\hat{r} = q\vec{E}$$

Coulomb's law for the field *E* due to *Q*:

$$\vec{E} = k\frac{Q}{r^2}\hat{r}$$

Example 1

What is the electric field strength at a distance of 10 cm from a charge of 2 μC?

$$E = \frac{kQ}{r^2} = \frac{(9\times10^{9})(2\times10^{-6})}{(10\times10^{-2})}$$

$$= \frac{18\times10^{3}}{10^{-1}} = 1.8\times10^{5}\ N/C$$

So a one-coulomb charge placed there would feel a force of 180,000 newtons.

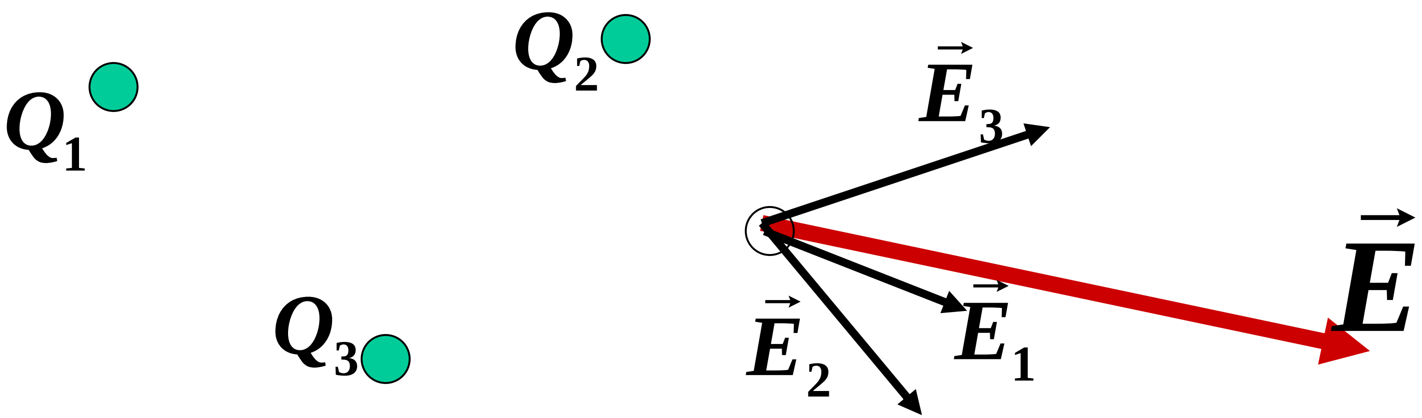
Q.22-1

A point charge Q is far from all other charges. At a distance of 2 m from Q, the electric field is 20 N/C. What is the electric field at a distance of 4m from Q?

1. 5 N/C
2. 10 N/C
3. 20 N/C
4. 40 N/C
5. 80 N/C

Adding fields

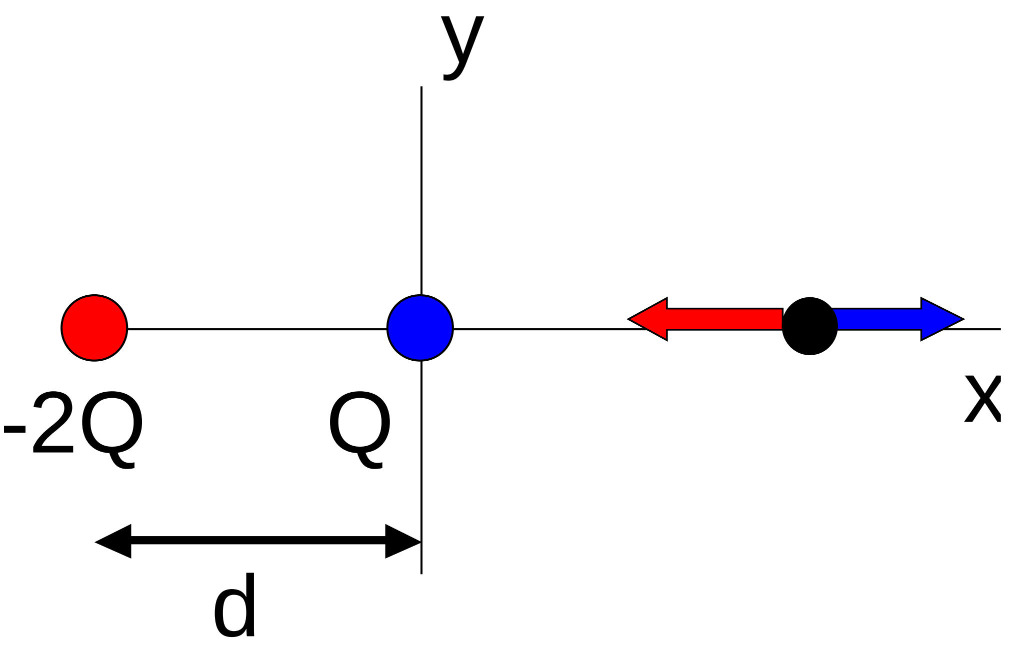
- Principle of superposition
- Electric fields due to different sources combine vector addition to form the one true total field.



$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \cdots$$

Example 2

1. **Find the electric field on the *x* axis.**



$$E_y = 0$$

$$E_x = E_1 - E_2 = \frac{kQ}{x^2} - \frac{2kQ}{(x+d)^2}$$

2. **Where will the field be zero?**

$$x + d = \sqrt{2}x$$

$$x = 2.4d$$

The shell theorems for E

In Chapter 13 we had the shell theorems for gravity

In Chapter 21 (p. 567) the shell theorems for electrostatics were stated.

In Chapter 23 (p. 618) they will be proven.

But we can easily understand them now from our knowledge of electric field lines.

The shell theorems for gravity

Given a **uniform spherical shell** of mass:

(1) The field outside is the same as if all the mass were concentrated at the center.

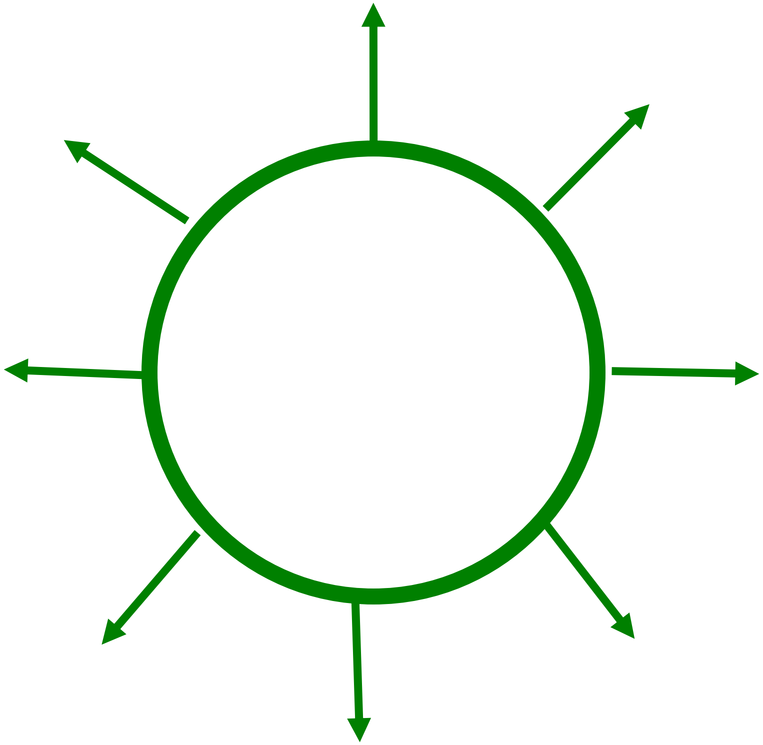
(2) The field inside the shell is zero.

(These theorems for gravity are given in Chapter 14.)

(Newton's headache!)

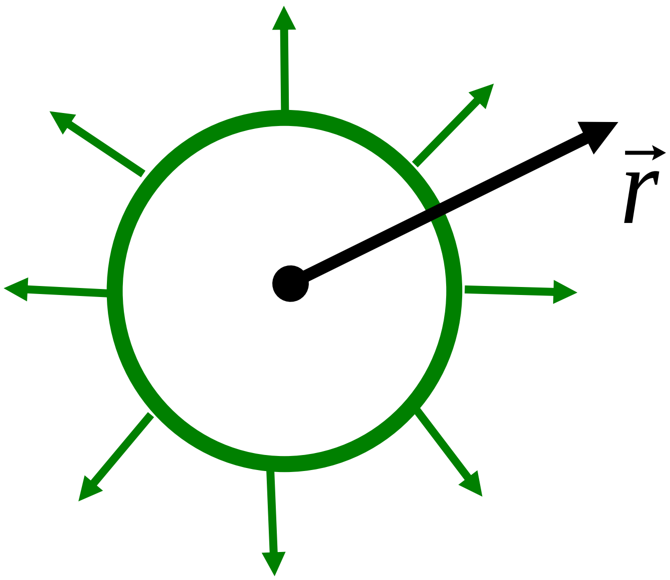
Prove true also for electric field

Use our knowledge of electric field lines to draw the field due to a spherical shell of charge:

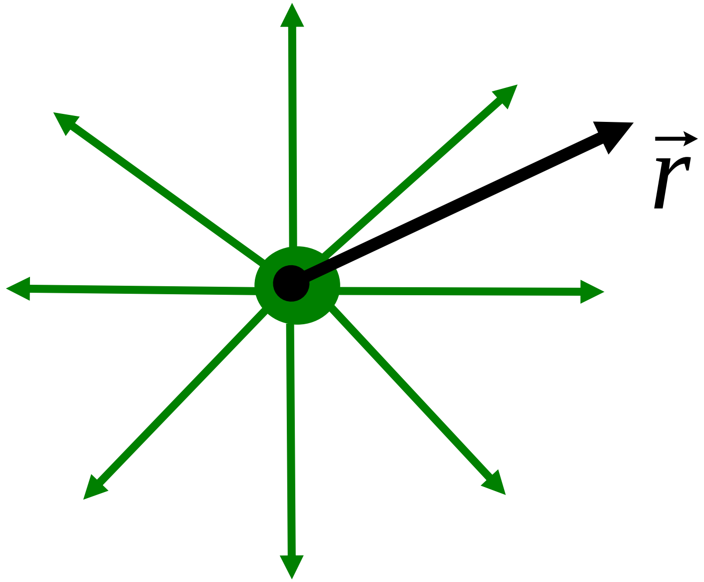


There is no other way to draw lines which satisfy all 3 properties of electric field lines, **and are also spherically symmetric**. **Notice that both shell theorems are obviously satisfied.**

Fields at $\vec{r}$ are the same!



Q spread over shell

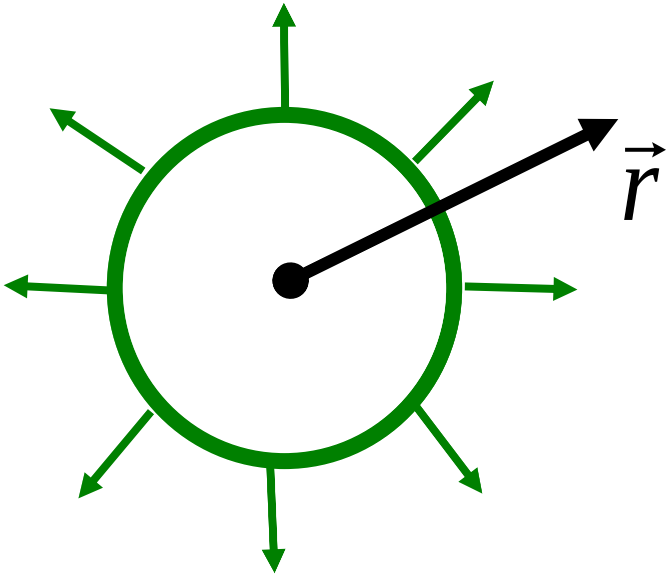


point Q at center

PROOF: **(1)** Spherical symmetry

(2) Fields far away must be equal

Useful result for spherical symmetry



Field outside a sphere of total charge Q is radially outward with magnitude

$$E = \frac{kQ}{r^2}$$

Q.22-2 **A shell of uniform charge attracts or repels a charged particle that is outside the shell as if all the shell's charge were . . .**

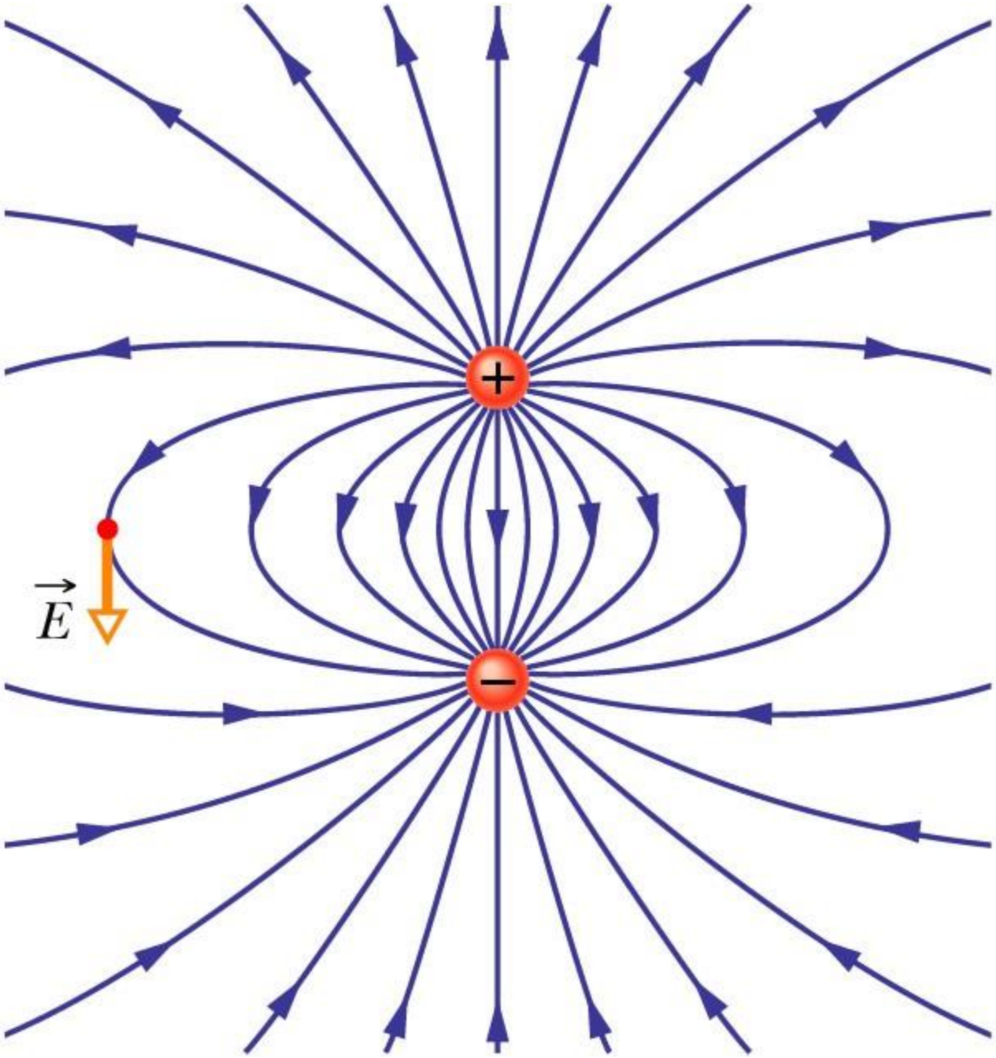
1. **Concentrated at the center.**
2. **Concentrated at the point closest to the particle.**
3. **Concentrated at the point opposite the particle.**
4. **Zero.**

Electric Dipole

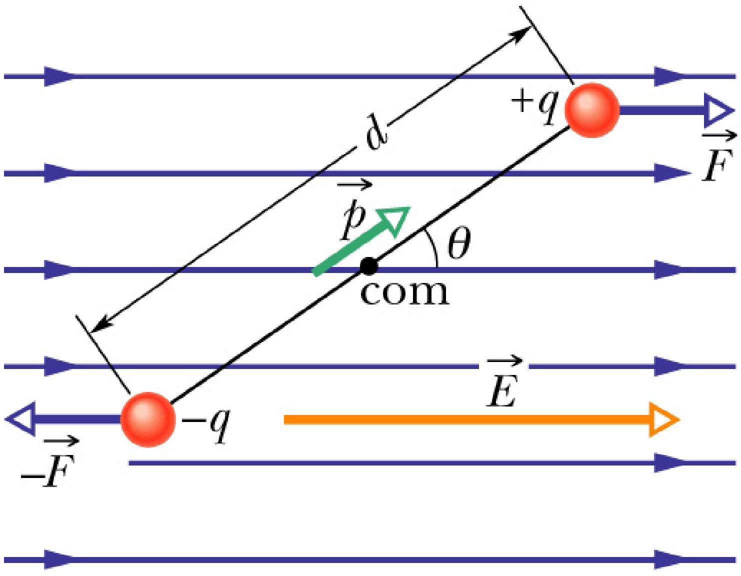
- The combination of two charges of equal but opposite sign is called a dipole.
- If the charges +q and –q are separated by a distance d, then the ***dipole moment*** $\vec{p}$ is defined as a vector pointing from –q to +q of magnitude $p = qd$.

$-q \quad \vec{p} \quad +q$

Electric Field Due to a Dipole



Torque on a Dipole in a Field



$$\tau = 2 \times F \times (\frac{d}{2}\sin\theta) = qE \times d\sin\theta = pE\sin\theta$$

$$\vec{\tau} = \vec{p} \times \vec{E}$$

Chapters 22, 23: The Electric Field

NOTE!

If you have urgent questions, please send me e-mail with

- **Your name**
- **Section**

At vkarpov@physics.utoledo.edu

Review: Electric Fields

- **Definition of *electric field:*** $\vec{F} = q\vec{E}$

  SI unit: newton per coulomb (N/C)
- **Coulomb's Law for a point charge *Q*:**

  $F = kQq/r^2$ OR $E = kQ/r^2$
- **Principle of superposition (*vector* addition):**

$$\vec{E}_{net} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \cdots$$

Electric Field Lines

We visualize the field by drawing field lines.
These are defined by three properties:

- **Lines point in the same direction as the field.**
- **Density of lines gives the magnitude of the field.**
- **Lines begin on + charges; end on – charges.**

From these properties it is easy to see that

- **Field lines never cross**

Coulomb's Law for the Field

Coulomb's law for the *force* on *q* due to *Q*:

$$\vec{F} = k\frac{qQ}{r^2}\hat{r} = q\vec{E}$$

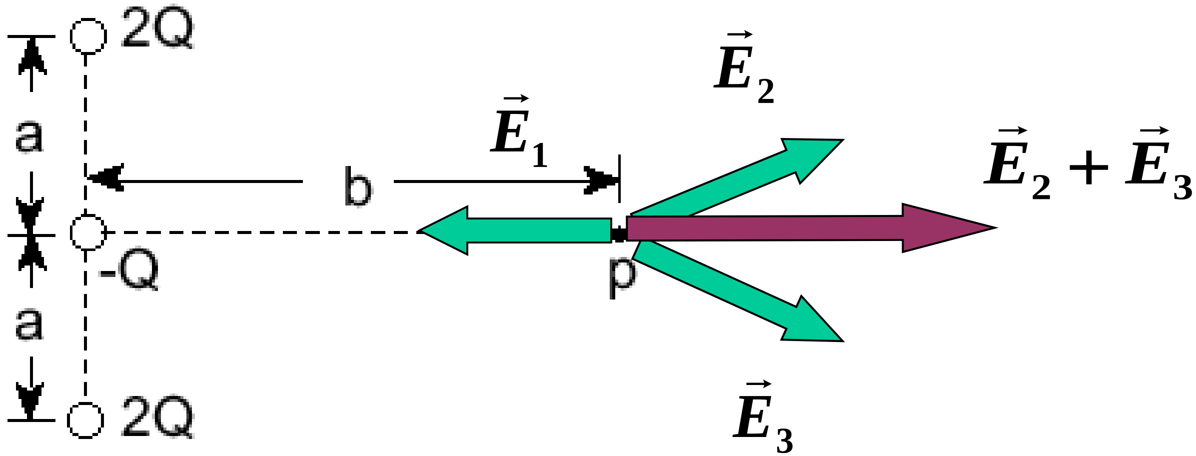
Coulomb's law for the *field* *E* due to *Q*:

$$\boxed{\vec{E} = k\frac{Q}{r^2}\hat{r}}$$

Example

Three point charges are placed on the *y* axis as shown. Find the electric field at point P on the *x* axis.

$$E_1 = \frac{kQ}{b^2}$$



$$E_2 = E_3 = k\frac{2Q}{a^2 + b^2}$$

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$$

$$E_y = 0$$

$$(\vec{E}_2 + \vec{E}_3)_x = 2E_2 \cos\Theta = 2E_2 b / \sqrt{a^2 + b^2}$$

$$E_x = (\vec{E}_2 + \vec{E}_3)_x - E_1 = 4kQb/(a^2 + b^2)^{3/2} - kQ/b^2$$

Q.22-1 **What is the SI unit for the electric field?**

1. **Newtons**
2. **Coulombs**
3. **Newtons per Coulomb**
4. **Newtons per meter**
5. **Coulombs per meter**

Q.22-1

What is the SI unit for the electric field?

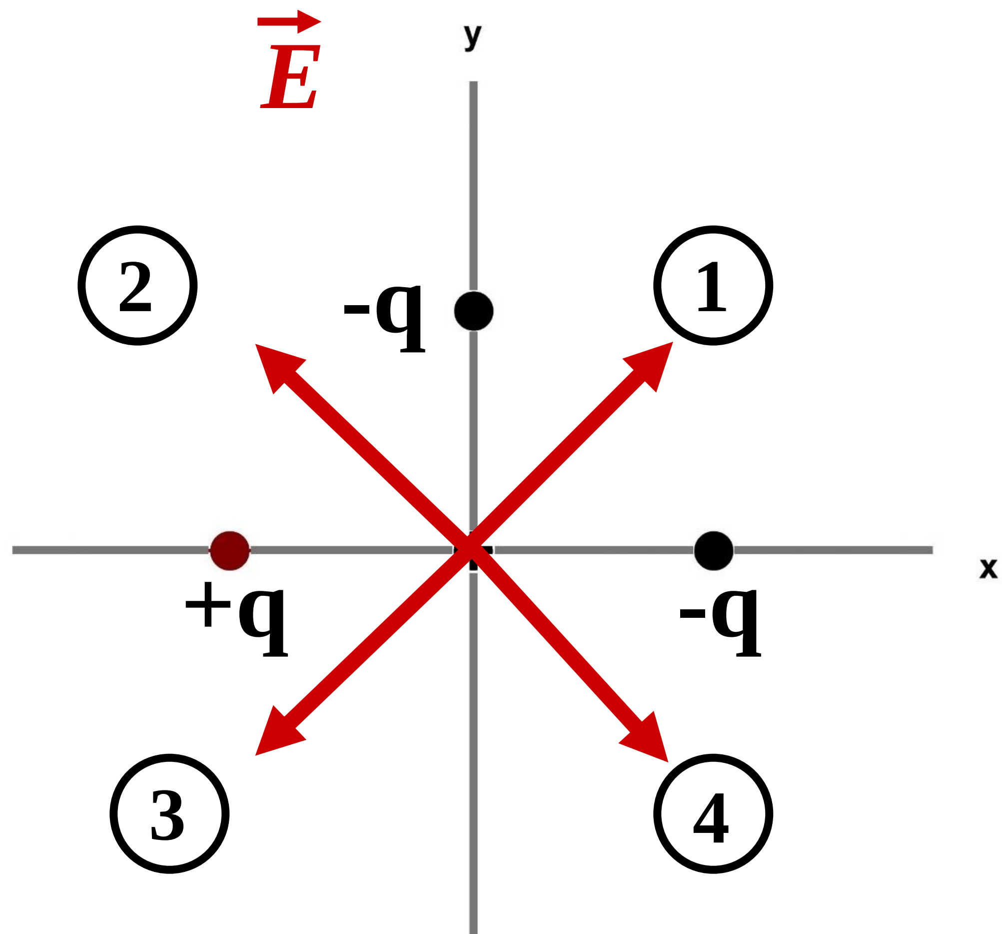
Field is force per unit charge:

$$\vec{F} = q\vec{E}$$

1. Newtons
2. Coulombs
3. Newtons per Coulomb
4. Newtons per meter
5. Coulombs per meter

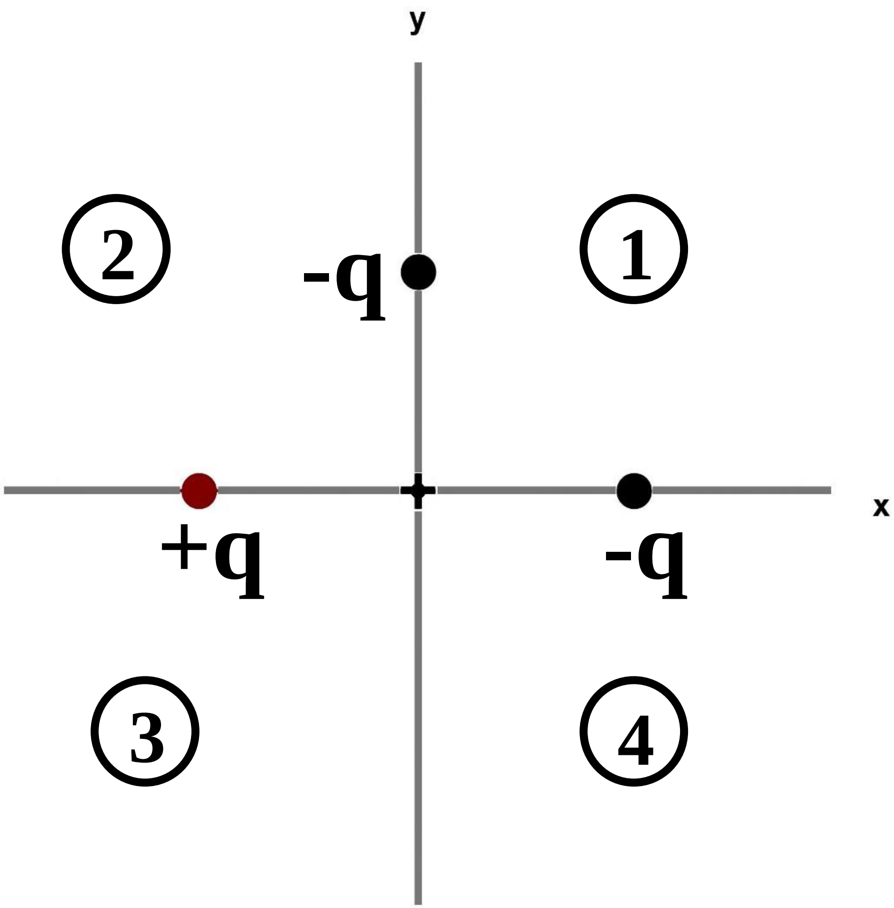
Three charges (one + and two -) are placed on the *x* and *y* axes as shown. What is the *approximate direction* of the electric field *at the origin*? Will it be pointing toward point 1, 2, 3, or 4?

Q.22-2



Q.22-2

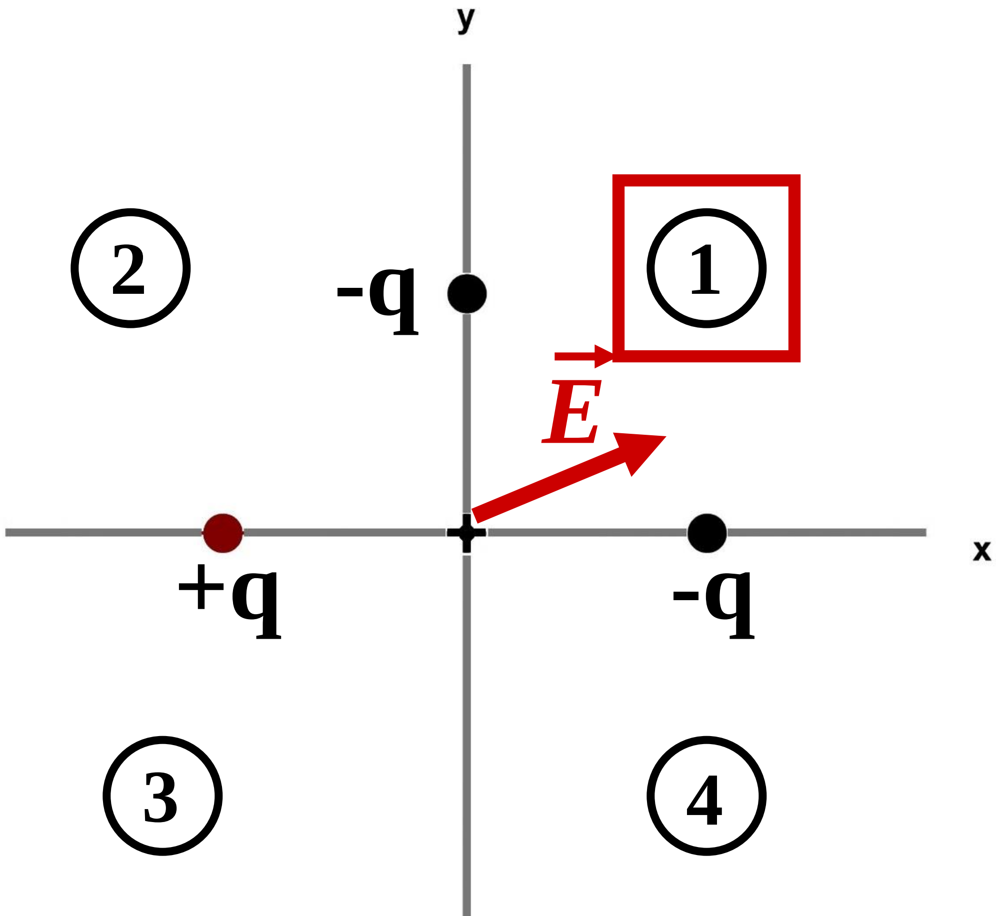
1. (1)
2. (2)
3. (3)
4. (4)



Q.22-2

Three charges (one + and two -) are placed on the *x* and *y* axes as shown. What is the *approximate direction* of the electric field *at the origin*? Will it be pointing toward point 1, 2, 3, or 4?

Solution. Imagine a positive test charge placed at the origin. It will be atracted to the –q charges and repelled by the +q charge.



Linear charge distribution

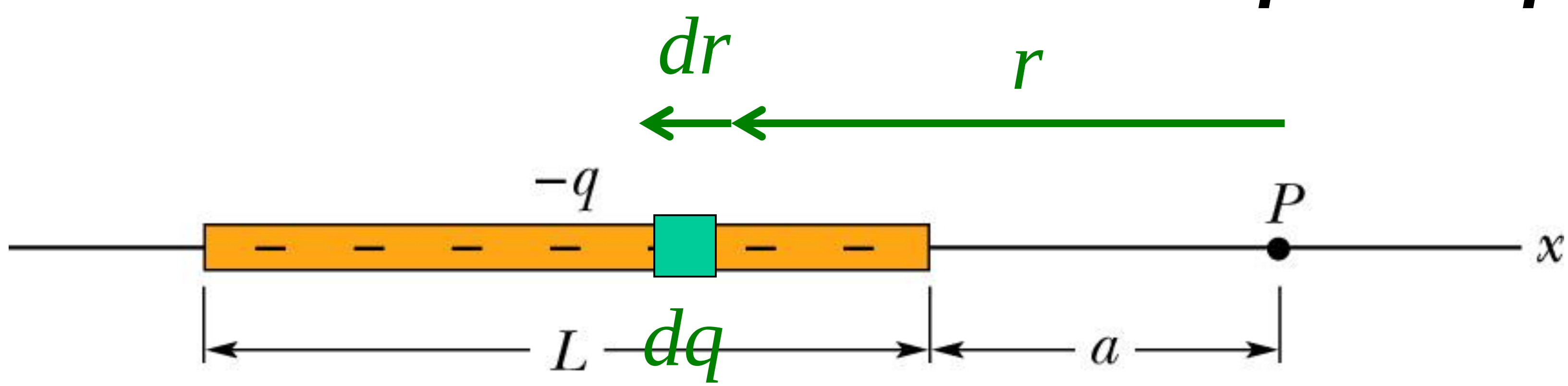
- ***Linear charge density*** **= charge per unit length**
- **If a rod of length 2.5 m has a *uniform* linear charge density λ = 3 C/m, then the total charge on the rod is (2.5 m) × (3 C/m) = 7.5 C.**
- **If a rod of length L carries a *non-uniform* linear charge density λ(x), then adding up all the charge produces an *integral:***

$$Q = \int_a^b dq = \int_a^b \lambda(x)\, dx$$

Example: Ch. 22 Prob. 32

Find the electric field at point P.

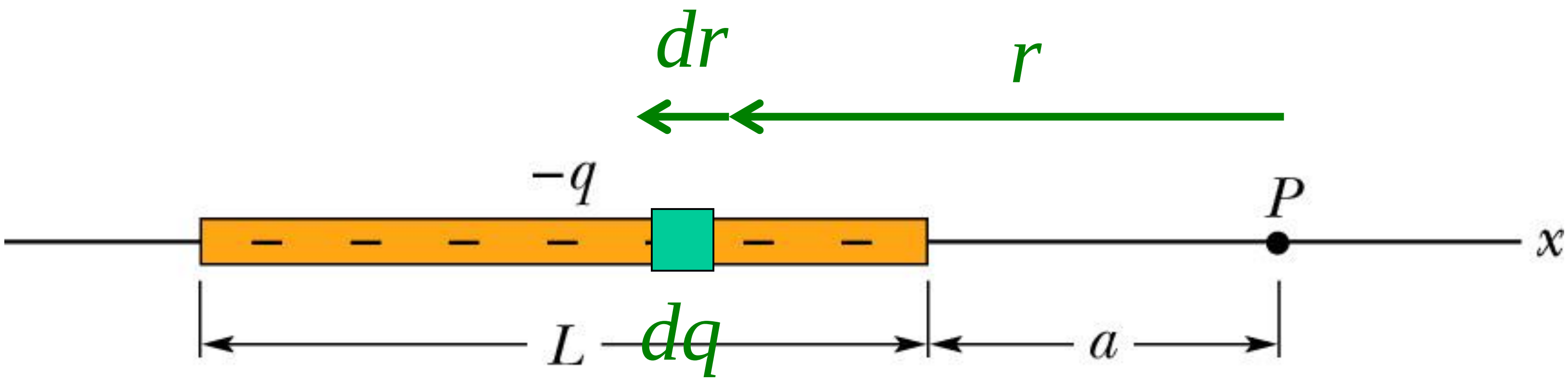
$$\text{Let } \lambda = -q/L. \quad \text{Then } dq = \lambda\, dr, \quad dE = k\frac{dq}{r^2} = k\lambda\frac{dr}{r^2}$$



$$E = \int dE = k\lambda \int_a^{a+L} \frac{dr}{r^2} = k\lambda\left[-\frac{1}{r}\right]_a^{a+L} = k\lambda\left(\frac{1}{a} - \frac{1}{a+L}\right)$$

Suppose density is not uniform

$$Still\ true\ that\ dq = \lambda\, dr, \quad dE = k\frac{dq}{r^2} = k\lambda\frac{dr}{r^2}$$



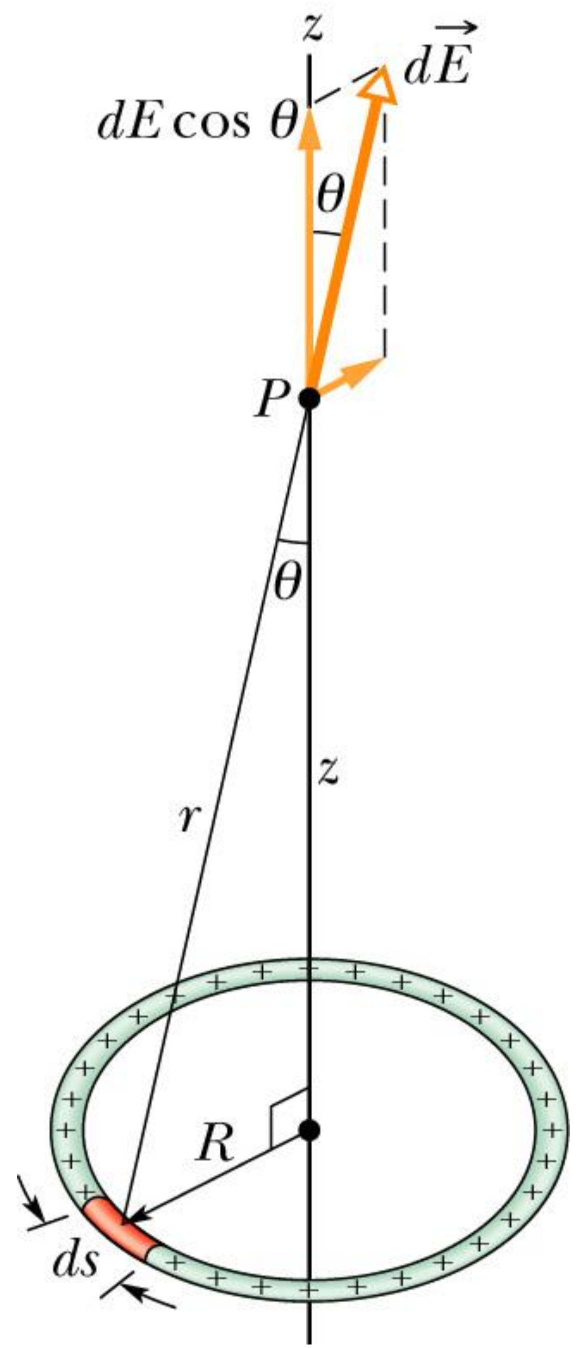
$$E = \int dE = k\int_a^{a+L} \lambda(r)\frac{dr}{r^2}$$

Field of a charged ring

Uniform linear charge density so $dq = \lambda ds$ and $dE = kdq/r^2$

By symmetry, $E_x = E_y = 0$ and so $E = E_z$ = sum of all dE_z, and $dE_z = \cos\theta\, dE$.

$$E = \int \frac{k\cos\theta\, dq}{r^2} = k\cos\theta \frac{Q}{r^2}$$

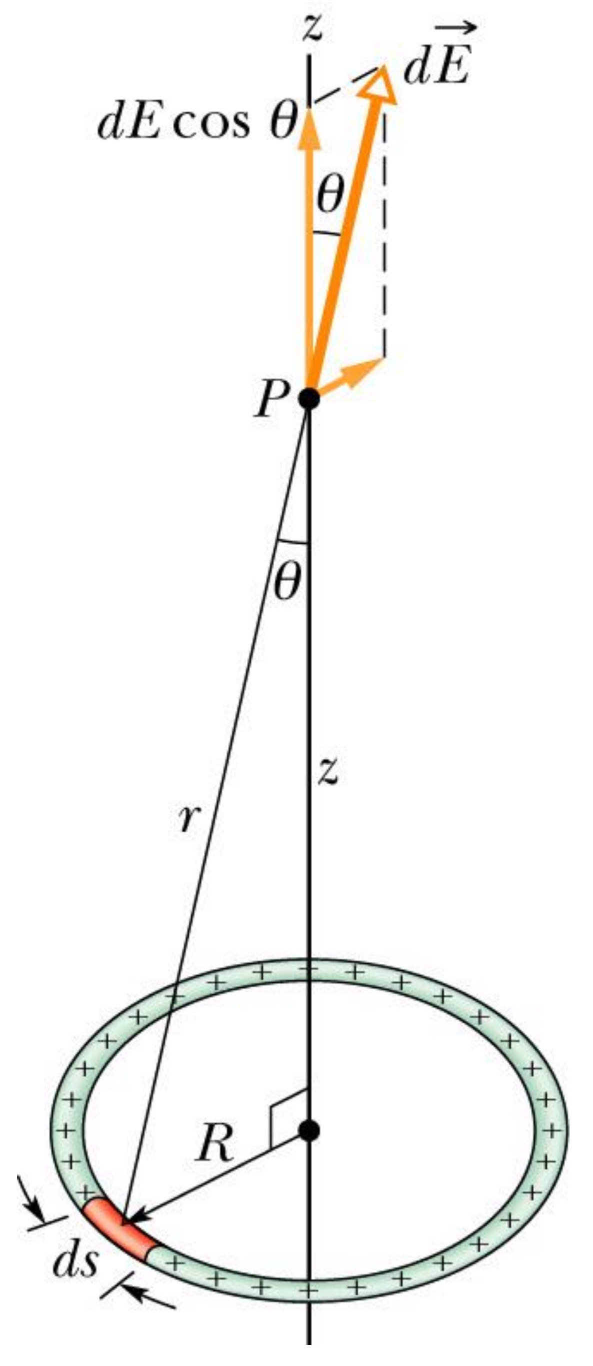


Charged ring continued

$$E=\int \frac{k\cos\theta\; dq}{r^2}=k\cos\theta\,\frac{Q}{r^2}$$

But $\cos\theta = z/r$ *so*

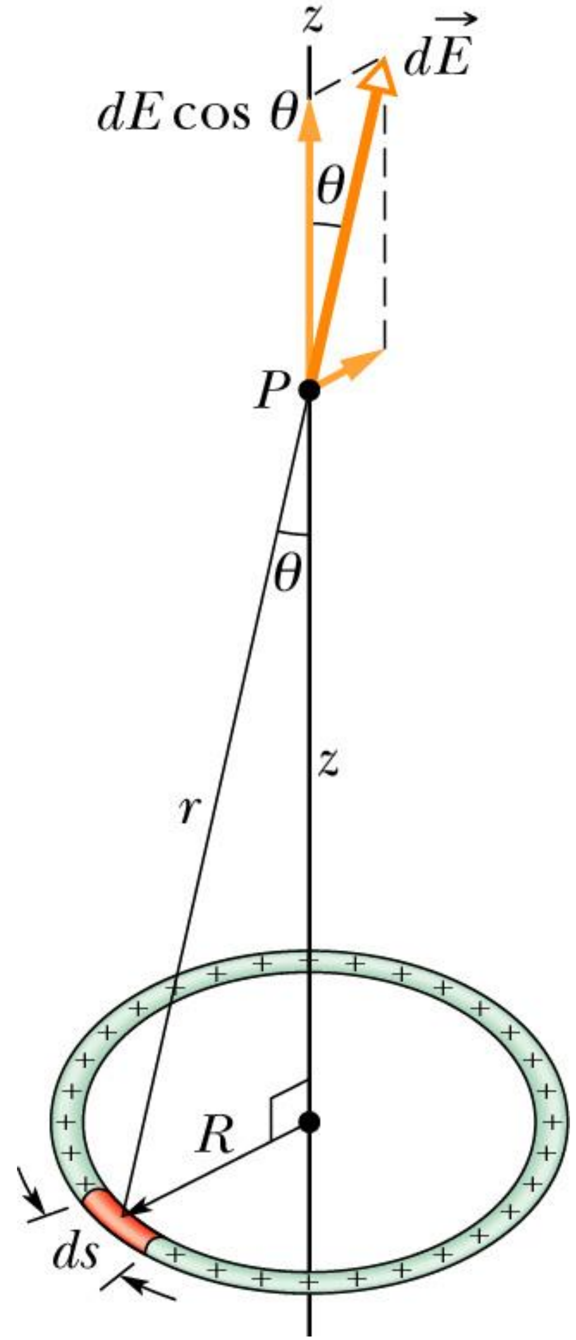
$$E=\frac{kQz}{r^3}=\frac{kQz}{(z^2+R^2)^{3/2}}$$



Charged ring result

$$E = \int \frac{k\cos\theta\, dq}{r^2} = k\cos\theta \frac{Q}{r^2}$$

$$= \frac{kQz}{r^3} = \frac{kQz}{(z^2 + R^2)^{3/2}}$$



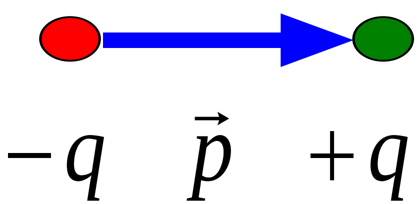
NOTE: This is a good example of a special result, which is the answer to an *example problem*, not a fundamental principle to be memorized. It is the *process* we are supposed to be learning, *not* the result!

NOTE

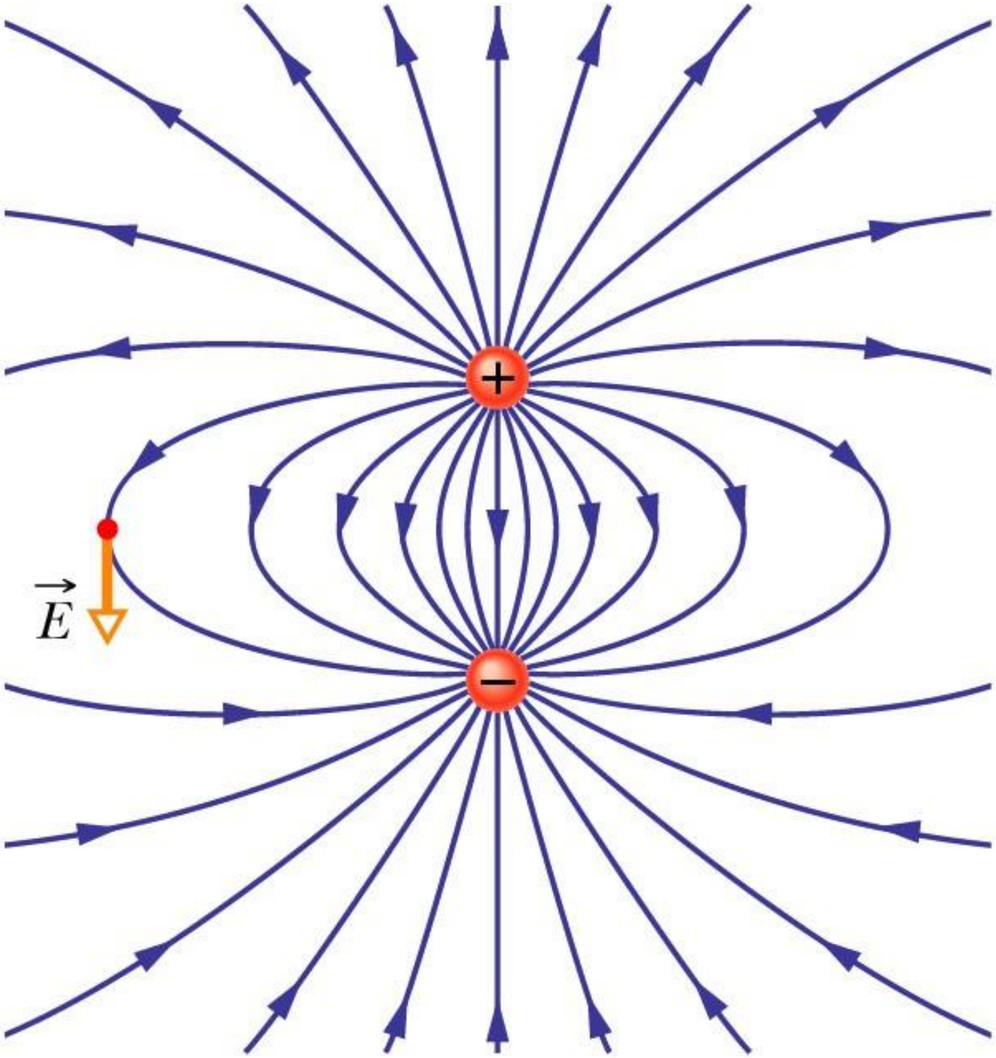
This result for the field on the axis of a charged ring can be derived more easily in Chapter 24 using the idea of ***electric potential.***

Electric Dipole

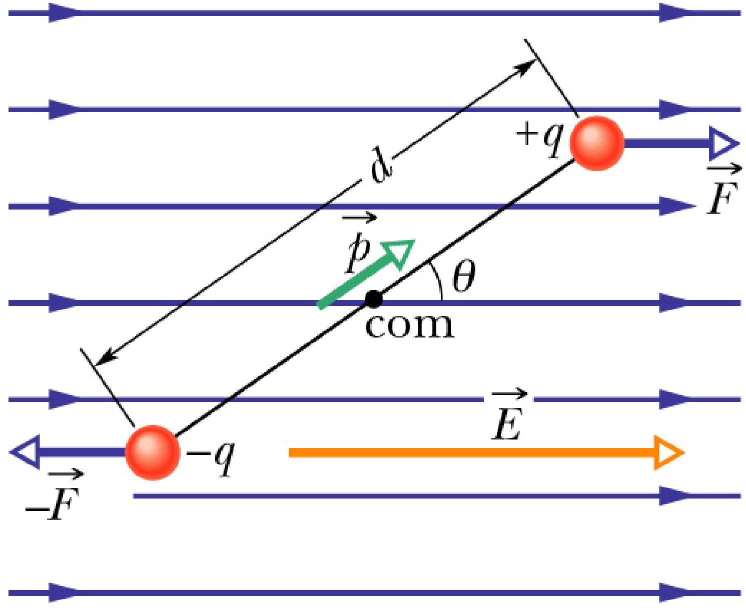
- The combination of two charges of equal but opposite sign is called a dipole.
- If the charges +q and –q are separated by a distance d, then the ***dipole moment*** $\vec{p}$ is defined as a vector pointing from –q to +q of magnitude $p = qd$.



Electric Field Due to a Dipole



Torque on a Dipole in a Field



$$\tau = 2 \times F \times \left(\frac{d}{2}\sin\theta\right) = qE \times d\sin\theta = pE\sin\theta$$

$$\vec{\tau} = \vec{p} \times \vec{E}$$

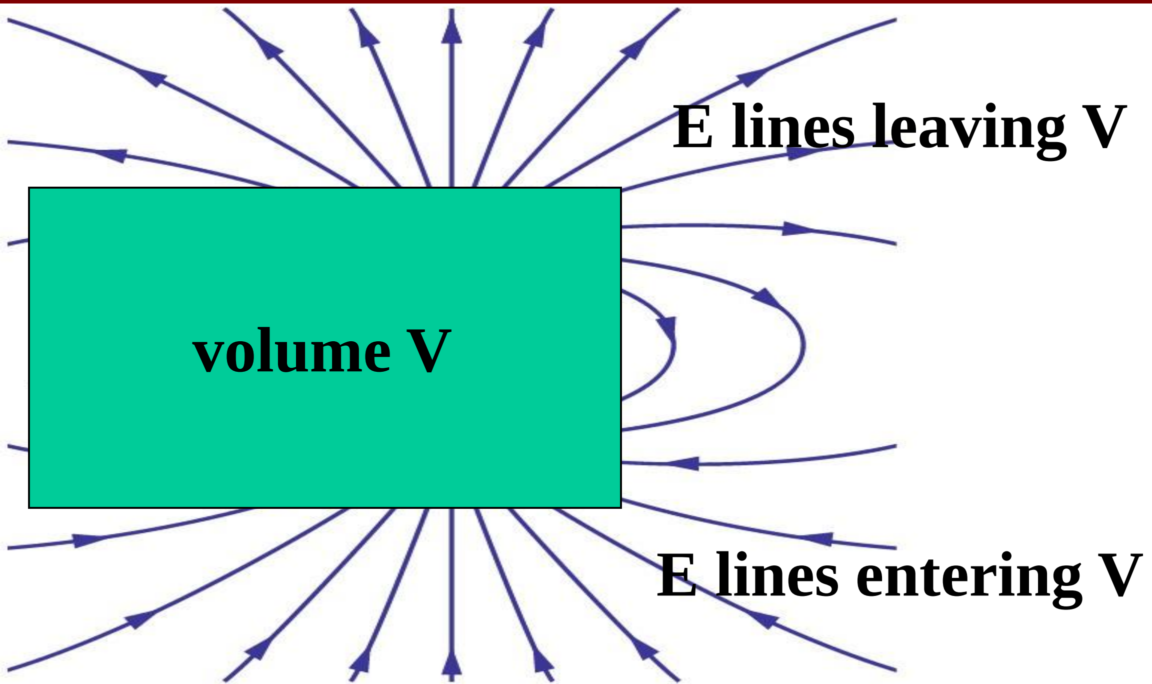
Gauss's Law

- **Gauss's Law is the first of the four *Maxwell Equations* which summarize all of electromagnetic theory.**

- **Gauss's Law gives us an *alternative* to Coulomb's Law for calculating the electric field due to a given distribution of charges.**

Gauss's Law: The General Idea

The net number of electric field lines which leave any volume of space is proportional to the net electric charge in that volume.



Flux

The ***flux Φ*** of the ***field E*** through the ***surface S*** is **defined** as

$$\Phi = \int_S \vec{E} \cdot d\vec{A}$$

The **meaning** of flux is just the **number of field lines** passing through the surface.

Best Statement of Gauss's Law

The outward flux of the electric field through any closed surface equals the net enclosed charge divided by ε_0.

Gauss's Law: The Equation

$$\oint_S \vec{E} \cdot d\vec{A} = Q_{enc} / \varepsilon_0$$

- S is any **closed** surface.
- Q_{enc} is the **net** charge **enclosed** within S.
- dA is an element of area on the surface of S.
- $d\vec{A}$ is in the direction of the **outward normal.**

$$\varepsilon_0 = 8.85 \times 10^{-12} \ SI \ units$$

Chapters 22, 23: The Electric Field

- **Previous Homework:**
  - Read Chapter 22
  - Do Ch. 22 Questions 3, 5, 7, 9
  - Do Ch. 22 Problems 5, 19, 24, 34